

Determination of Optimal Flood Control Levels in a Multi-reservoir System

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Abstract

In this study, a water resources system with multi-objective and multiple reservoirs is described. Optimization model used to the technique of dynamic programming with successive approximation (DPSA) for short term planning is established on this system. The Ceyhan River Basin is selected for application in the model. In the model, used the flow data of the flood hydrograph with design-objective (Probable maximum flood (PMF) hydrograph with 100-year-frequency), minimization of the peak value of the flow data released from reservoirs is achieved, and inflows released from one reservoir to other reservoir are carried by routing with kinematic wave approach in river. In the minimization, objective function is three stages, related to the maximum safe discharge for downstream and the additional control volume aimed to flood control. In the result of this optimization process, optimal flood control levels are obtained monthly.

Keywords: Multi-reservoir Systems, Flood Control, Dynamic Programming with Successive Approximation.

1. Introduction

Water resources planning are a multi-dimensional and complicated process. In the short term planning of multi-reservoir and multiple purpose systems, optimization and simulation techniques may be used in combination to obtain solutions which take into account the stochastic nature of the hydrological events like floods, where the risks must be reduced to a minimum and produce optimal benefits under certain risks.

Maximum safe discharge limit for the downstream of the reservoir of the total discharge released from reservoir is presented, while the minimization of the peak value of the flood hydrograph with design-objective entered to system for a definite month is described as objective function. In spite of additional flood control volume added on the normal operational level, if a solution proved this limit is not achieved, the solution purposed minimization of the peak value of the discharge data with flood event released from reservoir by setting free of this limit is repeated. Then, by reducing from the operational levels obtained to the additional flood control volume, the flood control levels are determined. In this case, proving of the reservoir safe is prior, and the maximum discharge released to downstream has been indicated to "a necessary damage value" [23]. Furthermore, safe operational levels produced optimal benefit for each month are obtained.

There have been a lot of studies for flood control and management. Most of these studies are generally taken part in the real-time optimal operation, [12 and 13]. It was utilized the same approach to develop operating rules for a water-supply reservoir during floods [14]. A methodology employing recursive linear programming (LP) to establish the optimal real-time operation of a network of flood control reservoirs was formulated [32]. A methodology to formulate flood control operating rules taking into account the value of flood warning time and increased damage due to duration of flooding was presented, in addition to the traditional considerations. There were parallel to those adopted in this study [1]. The Saint Venant Equations linearizated according to completely dynamic flow routing to the real-time optimal

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control model for reservoir system were explicitly included [29]. a methodology for solving the combined problem of real-time forecasting and short-term (daily) operation of a multireservoir system during floods was described [30]. A method for determining probabilistic release rules using Monte Carlo optimization was developed [31]. A methodology to determine reservoir operation policies based on explicit risk consideration was proposed [10]. An implicit stochastic optimization model for building seasonal rule curves for multipurpose multi-reservoir systems was formulated [16]. An atypical flood regulation approach was presented [5]. A stochastic stream flow model was used to develop a risk methodology for real-time reservoir flood operation. A FRB model to derive operation rules for a multipurpose reservoir in Oklahoma was used [27]. A detailed discussion and formulation of reservoir simulation models and optimization techniques including the emerging heuristic programming methods that utilize fuzzy logic, neural networks, and genetic algorithms was presented [12]. GA's to optimize the parameters in the fuzzy inference rules of a flood control reservoir model was utilized [3]. The simulation approach to derive operating rules for a multipurpose reservoir system in India was adopted. Monthly storage targets for conservation purposes were first defined for all reservoirs [11]. Linear programming for flood control in the Iowa and Des Monies Rivers was applied [17]. An FRB model along with a decision support system (DSS) to facilitate real-time flood control operations was used [4]. A reservoir operation support system based on fuzzy and neural network systems was developed [8]. It were applied to the routing coefficient method for real-time flood control procedures in the river basin, in Core [26].

Wave motion in the kinematic wave approach is described with continuity equation due to neglect the acceleration and the pressure terms in the St Venant equations. Firstly, the Saint-Venant Equations with one of the hydraulics and fluids mechanics base principles are revealed on describing the open canal with one dimension [22]. Then, the various studies were produced for the explicit solutions of these equations between 1953 and 1969 dates. These; It has been indicated that the flood wave motion was approximately resembled with kinematic wave motion in the river bed [15]. Definitions of the kinematic wave approach are mathematically revealed [9, 33, 34 and 35]. It has been related that the flow hydrograph in the river bed was estimated by used the kinematic wave approach [2]. The kinematic wave approach in order to process between rainfall and flow is used [7, 20 and 28]. The flow by used this approach was studied on carrying. Moreover, the wave motion in the water surface was contributed to estimate by used the kinematic wave equations [18 and 19].. It has been indicated to applicability of the kinematic wave and diffusion approaches [21]. It has been suggested to the kinematic wave approach for the superficial water [24 and 25]. It was described and applied to the kinematic wave routing technique in the HEC-1 simulation [6].

In this study, a water resources system is described, taking into consideration flood control, are developed using this system. In the model, an optimization technique using DPSA is used. In the optimization model for flood/ control, the peak values of the flow data released from reservoirs are minimized and inflows released from one reservoir to another reservoir are carried using the kinematic wave approach. In the minimization, the objective function has three stages, related to the maximum safe discharge downstream and the additional flood control levels of reservoirs. The model is applied to a water resource system with multi-reservoirs in the Ceyhan River Basin, Turkey.

2. Mathematical model of the system

Mathematical model of the system for the short term planning is based on hourly time scale. The system may be represented as a series of reservoirs, each with a storage capacity, inflows from upstream and releases to downstream in Figure 1. Inflows released from one reservoir to other reservoir are carried with kinematic wave approach in river. Taking i = 1, 2,..., N: Number of reservoirs, t= 1, 2,..., M: number of time intervals (M=12 months) and f= 1, 2,..., MM: number of time intervals (MM=720 hours), the basic constituent equations of the system are the water balance relations of each reservoir for each time interval, (for all i, t and f)

$$(S_{i,f+1})_t = (S_{i,f})_t + (F_{i,f})_t + (Q_{i-1,f})_t + (R_{i-1,f})_t - (Q_{i,f})_t - (R_{i,f})_t - (L_{i,f})_t$$
(1)

where $(S_{i,f})_t$: Water stored in reservoir i at time periods t (month) and f (hour), $(F_{i,f})_t$: Inflow into reservoir i at time periods t and f from its sub-drainage area, $(Q_{i,f})_t$: Water released for energy production from reservoir i at time periods t and f, $(R_{i,f})_t$:Spilled water from reservoir i at time periods t and f, $(L_{i,f})_t$: Water losses by evaporation and seepage from reservoir i at time periods t and f.

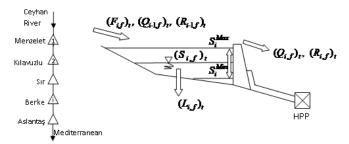


Figure 1. Operational variables in reservoir i at time period t (month) and f (hour)

In the DPSA formulation, the above equations represent the stage transformations equations where the time periods (f) are the stages and the storage levels at each reservoir ($(S_{i,f})_t$) are the states. Therefore, the releases from the reservoirs ($(Q_{i,f})_t$, $(R_{i,f})_t$) appear as the basic decision variables. System constraints are related to the storage capacity, the releases for energy production, the spill water and the total water releases.

1. Constraints on the storage capacities are two forms; the first form is given in Eq. (2) without additional flood control volume.

$$S_{i}^{Min} \leq (S_{i,f})_{t} \leq S_{i}^{Max}$$

$$\tag{2}$$

The second form is given in Eq. (3) with additional flood control volume.

$$S_i^{Min} \le (S_{i,f})_{\mathfrak{t}} \le S_i^{Max^*}$$
(3)

where, $S_i^{Max^*}$ is the added maximum storage capacity in the reservoir i and it is given in Eq. (4).

$$S_i^{Max*} = S_i^{Min} - (S_t^{Maks} - S_{i,t}^{Nor})$$
(4)

where $S_{i,t}^{Nor}$ is the monthly normal operational storage capacity of the reservoir i at a time period t. This storage capacity is obtained from optimal operation for long term planning. $S_{i,t}^{Nor}$ - S_i^{Min} is also the additional flood control volume of the reservoir i at a time period t.

1. Constraints on the releases for energy production

$$Q_i^{Min} \leq (Q_{i,f})_t \leq Q_i^{Max}$$
⁽⁵⁾

where Q_i^{Min} and Q_i^{Max} are the minimum and maximum releases for energy production from reservoir i. Here, Q_i^{Max} is limit with installed power.

2. Constraints on the spill water

$$0 \le (R_{i,f})_{t} \le R_{i}^{Max} \tag{6}$$

where R_i^{Max} is the maximum spillway capacity for reservoir i; 3. Constraints on the total water releases

$$D_i^{Min} \le (Q_{i,f} + R_{i,f})_t \le D_i^{Max}$$
(7)

where D_i^{Min} is the minimum release for pollution control, navigation etc. D_i^{Max} is the maximum safe discharge for downstream of reservoir i (with flood control).

3.Optimization model

A technique based on DPSA optimization was used in the optimization models. In the DPSA formulation, the above equations represent the stage transformations equations, where the time periods (t) are the stages and the storage levels at each reservoir $((S_{i,f})_t)$ are the states. Thus, the releases from the reservoirs $((Q_{i,f})_t \operatorname{and} (R_{i,f})_t)$ appear as the basic decision variables. However, it must be noted that the spilled water release $(R_{i,f})_t$ will only take place when the storage and turbine release capacity constraints of the system are about to be violated; otherwise it will be zero. Thus, $(R_{i,f})_t$ is a dependent variable, and the real decision variable is $(Q_{i,f})_t$.

The objective function has three stages, related to the maximum safe discharge downstream and the additional flood control volume. If the solution for the first stage is not realized, the transition from the first stage to the second stage can be continued and, if not, the transition from the second one to the third one can be performed:

1. For minimization of the peak value of the flow data released from reservoirs with the maximum safe discharge downstream, but without the additional flood control volume, using the flow data of the flood hyetograph with design-objective, the objective function in the first stage may be stated as

$$Min[Max(Q_{i,f} + R_{i,f})_t]$$
(7)

The solution may be evaluated by DPSA

1. In the second stage for minimization of the peak value of the flow data released from reservoirs with the maximum safe discharge downstream and the additional flood control volume, using the flow data of the flood hyetograph with design-objective, the objective function may be stated in Eq. (7). When the additional flood control volumes are used, the constraints of the storage capacity for flood control volumes are

$$S_i^{Min} \le (S_{i,f})_{\mathfrak{t}} \le S_i^{Max^*} \tag{8}$$

$$S_{i}^{Max^{*}} = S_{i}^{Max} + (S_{i}^{Nor} - S_{i,t}^{Min})$$
(9)

where $S_i^{Max^*}$ is the maximum additional storage capacity of the reservoir for flood control. 3. In the third stage for minimization of the peak value of the flow data released from reservoirs with the re-determined maximum safe discharge downstream and the additional flood control volume (this re-determined maximum safe discharge for downstream is

flood control volume (this re-determined maximum safe discharge for downstream is calculated relative to the damage level accepted in river), using the flow data of the flood hyetograph with design-objective, the objective function may be stated as in Eq. (7).

As a result of this process, the operation levels of the flood control are determined by reducing the additional flood control volumes from operation storage levels obtained for each reservoir and month. The smallest operation levels of this flood control for each reservoir and month are assigned as optimal flood control levels. The optimal flood control levels are not greater than the maximum operation levels of the reservoirs (with completed to long and short-term planning). If the optimal flood control level is greater/smaller than the normal operation level of reservoir, the difference between the optimal flood control level and the normal operation level is determined. The optimal flood control level is re-obtained by reducing from normal operation level according to this difference.

4. Methods

4.1. DPSA optimization technique

The purpose of DPSA is to divide the problem of dynamic programming with multi-decision variables to the sub-problems of dynamic programming with one-decision variable in each one and, thus, to solve the problem while taking decision variables one-by-one. The DPSA optimization technique has more advantage than other versions of dynamic programming in terms of reducing both the calculated time and the memory requirements of the computer. There are three variables in the DPSA: state, decision and stage variables. The team of these values related to some constrains of these variables is called the system politic. The criterion

determined the effect of this system politic is also described as the objective function. The schematic view of the state-decision-stage variables in the DPSA of a reservoir is shown in Figure 2.

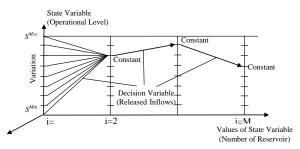


Figure 2. Schematic view of the state-decision-stage variables in the DPSA

4.2 Kinematic wave approach

In the kinematic wave approach, the wave motion consisting of the inflows released from reservoirs due to neglecting the acceleration and the pressure terms in St Venant equations is described with the continuous equation only. The curves of the arrived value and time to other reservoirs of the inflows released from reservoirs can be constituted using this approach. In the optimization process for short term planning, the total inflows released from the turbine/spillway of the reservoirs are limited by the total released capacity. Inflow values into other reservoirs of the total inflows released from reservoirs in this approach are added to the sub-drainage areas of reservoirs by taking into consideration the arrival times of the arrived inflow values

5. Application

A water resource system with multiple reservoirs in the Ceyhan Basin in Turkey was selected for application. There are five sequential reservoirs situated on the Ceyhan River (at the upper reaches of the Mediterranean) with energy production, flood control and irrigation-drinking water functions in the Ceyhan Basin. The system was originally planned for energy production only. Table 1 shows the basic characteristics of the five reservoirs in the system. The variations of the reservoir storage volumes with heights are given in Table 2.

Table 1. Basic characteristics of reservoirs in the system.

Reservoirs	Menzelet	Kılavuzlu	Sır	Berke	Aslantaş
Drainage Area (km ²)	8340	8486	12950	13222	14874
Elevations (m)	609.4 - 485.5	485.5 - 440	440 - 345	345 - 159	159 - 84
Installed Power (MW) (Upper limits)	124	54	273	510	138
Dam Height (m)	150,5	56	120	201	95
Maximum operational level (m)	609,4	485,5	440	345	159
Minimum operational level (m)	560,2	483,5	418,8	288,8	130
Maximum Volume (10 ⁶ m ³)	1950	74	1117,94	427	1990
Minimum Volume (10 ⁶ m ³)	533,6	69	451,3	119	530
HPP Elevation (m)	485,5	440	345	159	84

HPP: Hydroelectric Power Plant

Table 2.	Volume-height relationshi	ps for reservoirs

Reservoirs	Menzelet	Kılavuzlu	Sır	Berke	Aslantaş
а	30.898	24.182	45.328	76.157	9.5129

b	0.2998	0.4092	0.2067	0.2581	0.3947
(h=a.S ^b , h (m), S	$(10^7 \text{ m}^3))$				

Inflows into the reservoirs were obtained from the data from the flow gauging stations, correlating the drainage areas of the reservoirs and the gauging stations. Then, inflows into each reservoir from its own sub-drainage area were evaluated. The hourly inflows of the flood cases into reservoir from its sub-drainage area are shown in Figure 3

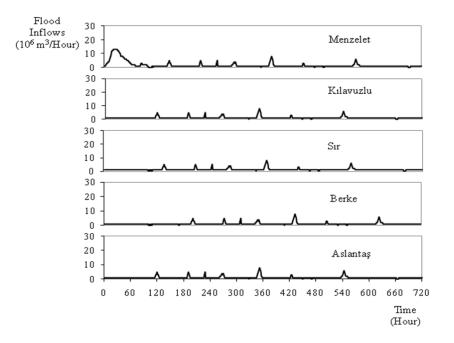


Figure 3. Hourly inflows of the flood case into the reservoirs from sub-drainage areas

For the DPSA solution technique $10x10^4$ m³ discrete volumes both for storages and releases have been used. Furthermore, the optimal operation during a flood case in reservoirs is started by supposing the required time is large enough to decrease the optimal operational levels for flood control. Furthermore, in the optimization process, inflows released from reservoirs to other reservoirs are routed using the kinematic wave approach. Table 3 shows the basic characteristics of the cross section of the Ceyhan River.

Tuble 5. The busic characteristics of the river and its cross section										
Reservoirs	L	Η	S	n	h	b	Α	Р	R	Q
	(m)	(m)			(m)	(m)	(m ²)	(m)	(m)	(m ³ /s)
Menzelet-Kılavuzlu	6820	123.9	0.0182	0.025	1.5	30	45	33	1.36	298
Kılavuzlu-Sır	35610	45.5	0.0013	0.025	2.5	48	120	53	2.26	295
Sır-Berke	4340	95	0.0219	0.025	1.4	30	42	32.8	1.28	293
Berke-Aslantaş	45610	186	0.0041	0.025	2	40	80	44	1.82	304

Table 3. The basic characteristics of the river and its cross section

6. Results

As a result of the optimization process, the operational levels for flood control of each reservoir are obtained. These levels are shown in Figures 4 for flood control. The largest variations are shown in the operational levels for the flood/drought control and management in the Menzelet and Aslantaş reservoirs, where the reason of these variations is the storage levels of the Menzelet and Aslantaş Reservoirs. These reservoirs are a controlling-managing

case of the multiple-reservoirs system. The optimal operational levels for flood control are given in Table 4.

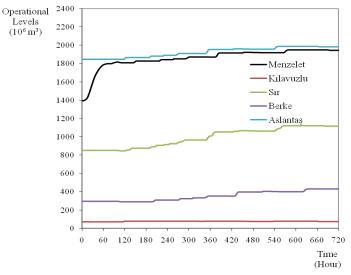


Figure 4. Operational levels for flood control in reservoirs

Table 4. Optimal operational levels for flood/drought control and management

Reservoirs	Menzelet	Kılavuzlu	Sır	Berke	Aslantaş	
$S_{i,t}^{Nor}$ (10 ⁶ m ³)	1670	80	1120	430	1990	
$S_{i,t}^{Op,Flood}$ (10 ⁶ m ³)	752	70	705	120	530	
S_i^{Min} (10 ⁶ m ³)	530	70	450	120	530	
S_i^{Maks} (10 ⁶ m ³)	1950	80	1120	430	1990	

In the optimization process, inflows released from one reservoir to another reservoir are carried using the kinematic wave approach in the Ceyhan River. The achieved inflows of the inflows released from reservoirs to other reservoirs are shown in Figure 5. The achieved times of the inflows released from reservoirs to other reservoirs are shown in Figure 6.

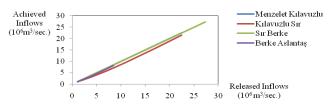


Figure 5. Achieved inflows of inflows released from reservoirs to other reservoirs

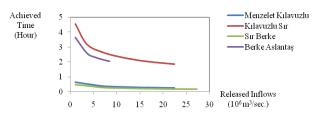


Figure 6. Achieved times of inflows released from reservoirs to other reservoirs

Conclusions

Mathematical model together with all of the variables of a water resources system consisted of reservoirs serially connected to each other in a river basin is described, established, and the model is successively applied to a water resources system in the Ceyhan River Basin. At the end of the optimization process, optimal flood control levels monthly are obtained from flood control levels for each reservoir. Optimal operation during a flood event is started by supposing the required time enough to go down the optimal flood control level. In this study, methods used as different from other studies in the optimization model are DPSA and kinematic wave approach. Results obtained from this process are evaluated below;

- Stored to the flood inflow data by multiple reservoir system, completely
- Controlled and managed of optimization process by reservoirs with large storage

Various analyses to be minimized the flood damage in downstream may be produced when the optimal operation is not the required time enough to go down the optimal flood control level. In addition to this, providing that flood damage is described as financial, analyses may be presented again.

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